Chapter 10

Section 10.1

Check Your Understanding, page 619:

1. *State:* Our parameters of interest are p_1 = true proportion of teens who go online every day and p_2 = true proportion of adults who go online every day. We want to estimate the difference $p_1 - p_2$ at a 90% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: n_1 = 799 is less than 10% of the population of teens and n_2 = 2253 is less than 10% of the population of adults. Large Counts: $n_1\hat{p}_1 = 799(0.63) = 503 \geq 10$, $n_1(1-\hat{p}_1) = 799(0.37) = 296 \geq 10$, $n_2\hat{p}_2 = 2253(0.68) = 1532 \geq 10$, and $n_2(1-\hat{p}_2) = 2253(0.32) = 721 \geq 10$. *Do:* The 90% confidence interval is:

$$(0.63-0.68) \pm 1.645\sqrt{\frac{0.63(0.37)}{799} + \frac{0.68(0.32)}{2253}} = -0.05 \pm 0.0324 = (-0.0824, -0.0176).$$

Conclude: We are 90% confident that between the interval from –0.0824 to –0.0176 captures the true difference in the proportion of U.S. adults and teens who go online every day.

Check Your Understanding, page 628:

State: We want to perform a test at the $\alpha = 0.05$ significance level of $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ where p_1 is the true proportion of children like the ones in the study who do not attend preschool that use social services later and p_2 is the true proportion of children like the ones in the study who attend preschool that use social services later. *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from two groups in a randomized experiment. Large Counts: The number of successes and failures in both groups are at least 10 (No preschool: 49 successes, 12 failures. Preschool: 38 successes, 24 failures).

Do: The proportions of those using social services in each group are $\hat{p}_1 = \frac{49}{61} = 0.8033$ and $\hat{p}_2 = \frac{38}{62} = 0.6129$. The pooled proportion is $\hat{p}_C = \frac{49+38}{61+62} = \frac{87}{123} = 0.7073$. The test statistic is

$$z = \frac{(0.8033-0.6129)-0}{\sqrt{\frac{0.7073(0.2927)}{61} + \frac{0.7073(0.2927)}{62}}} = 2.32$$ and the *P*-value is $P(Z > 2.32) = 0.0102$.

Conclude: Because the *P*-value of 0.0102 is less than α = 0.05, we reject H_0. There is convincing evidence that the true proportion of children like the ones in the study who do not attend preschool that use social services later is greater than the true proportion of children like the ones in the study who attend preschool that use social services later. In other words, children like those in this study who participate in preschool are less likely to use social services later in life.

Exercises, page 629:

10.1 (a) Because $n_1p_1 = 100(0.25) = 25$, $n_1(1-p_1) = 100(0.75) = 75$, $n_2p_2 = 100(0.35) = 35$, and $n_2(1-p_2) = 100(0.65) = 65$ are all at least 10, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.

(b) The mean is $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2 = 0.25 - 0.35 = -0.10$.

(c) Because $n_1 = 100$ is less than 10% of the first bag and $n_2 = 100$ is less than 10% of the second bag, the standard deviation is

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = \sqrt{\frac{0.25(0.75)}{100} + \frac{0.35(0.65)}{100}} = 0.0644.$$

10.2 (a) Because $n_1p_1 = 150(0.70) = 105$, $n_1(1-p_1) = 150(0.30) = 45$, $n_2p_2 = 150(0.50) = 75$, and $n_2(1-p_2) = 150(0.50) = 75$ are all at least 10, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal.

(b) The mean is $\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2 = 0.70 - 0.50 = 0.20$.

(c) Because $n_1 = 150$ is less than 10% of the students at school 1 and $n_2 = 150$ is less than 10% of the students at school 2, the standard deviation is

$$\sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = \sqrt{\frac{0.70(0.30)}{150} + \frac{0.50(0.50)}{150}} = 0.0554.$$

10.3 (a) The sampling distribution of $\hat{p}_C - \hat{p}_A$ is approximately Normal because $n_Cp_C = 50(0.30) = 15$, $n_C(1-p_C) = 50(0.7) = 35$, $n_Ap_A = 100(0.15) = 15$, and $n_A(1-p_A) = 100(0.85) = 85$ are all at least 10.

(b) The mean is $\mu_{\hat{p}_C-\hat{p}_A} = p_C - p_A = 0.30 - 0.15 = 0.15$.

(c) Because $n_C = 50$ is less than 10% of the jelly beans in the Child mix and $n_A = 100$ is less than 10% of the jelly beans in the Adult mix, the standard deviation is

$$\sigma_{\hat{p}_C-\hat{p}_A} = \sqrt{\frac{p_C(1-p_C)}{n_C} + \frac{p_A(1-p_A)}{n_A}} = \sqrt{\frac{0.3(0.7)}{50} + \frac{0.15(0.85)}{100}} = 0.0740.$$

10.4 (a) The sampling distribution of $\hat{p}_G - \hat{p}_D$ is approximately Normal because $n_Gp_G = 60(0.8) = 48$, $n_G(1-p_G) = 60(0.2) = 12$, $n_Dp_D = 75(0.4) = 30$, and $n_D(1-p_D) = 75(0.6) = 45$ are all at least 10.

(b) The mean is $\mu_{\hat{p}_G-\hat{p}_D} = p_G - p_D = 0.80 - 0.40 = 0.40$.

(c) Because $n_G = 60$ is less than 10% of all high school graduates and $n_D = 75$ is less than 10% of all high school dropouts, the standard deviation is

$$\sigma_{\hat{p}_G-\hat{p}_D} = \sqrt{\frac{p_G(1-p_G)}{n_G} + \frac{p_D(1-p_D)}{n_D}} = \sqrt{\frac{0.80(0.20)}{60} + \frac{0.40(0.60)}{75}} = 0.0766.$$

10.5 The Random condition is not met because these data do not come from independent random samples or two groups in a randomized experiment. Also, the Large Counts condition is not met because there were less than 10 successes (3) in the group from the west side of Woburn.

10.6 The Large Counts condition is not met because there were less than 10 successes (6) in the group wearing wrist guards.

10.7 The Large Counts condition is not met because there were less than 10 failures (0) in the treatment group, less than 10 successes (8) in the control group, and less than 10 failures in the control group (4).

10.8 The Large Counts condition is not met because there were less than 10 successes (0) in the microwave group.

10.9 (a) $\text{SE}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.26(1-0.26)}{316}+\frac{0.14(1-0.14)}{532}} = 0.0289$. If we were to take many random samples of 316 young adults and 532 older adults, the difference in the sample proportions of young adults and older adults who use Twitter will typically be 0.0289 from the true difference in proportions of all young adults and older adults who use Twitter.
(b) *State:* Our parameters of interest are $p_1 =$ true proportion of young adults who use Twitter and $p_2 =$ true proportion of older adults who use Twitter. We want to estimate the difference $p_1 - p_2$ at a 90% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. Random: The data come from two independent random samples. 10%: $n_1 = 316$ is less than 10% of all young adults and $n_2 = 532$ is less than 10% of all older adults. Large Counts: $n_1\hat{p}_1 = 316(0.26) = 82.16 \approx 82$, $n_1(1-\hat{p}_1) = 316(0.74) = 233.84 \approx 234$, $n_2\hat{p}_2 = 532(0.14) = 74.48 \approx 74$, and $n_2(1-\hat{p}_2) = 532(0.86) = 457.52 \approx 458$ are all at least 10.
Do: The 90% confidence interval is $(0.26-0.14) \pm 1.645\sqrt{\frac{0.26(1-0.26)}{316}+\frac{0.14(1-0.14)}{532}} =$ 0.12 ± 1.645(0.0289) = 0.12 ± 0.048 = (0.072, 0.168) *Using technology*: (0.073, 0.168).
Conclude: We are 90% confident that the interval from 0.073 to 0.168 captures the true difference in the proportions of young adults and older adults who use Twitter.

10.10 (a) From the data we find that $n_1 = 634$, $\hat{p}_1 = \frac{368}{634} = 0.580$, $n_2 = 567$, and $\hat{p}_2 = \frac{130}{567} = 0.229$, so $\text{SE}_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{0.580(1-0.580)}{634}+\frac{0.229(1-0.229)}{567}} = 0.0264$. If we were to take many random samples of 634 young blacks and 567 young whites, the difference in the sample proportions of young blacks and young whites who listen to rap every day will typically be 0.0264 from the true difference in proportions of all young blacks and young whites who listen to rap every day.

(b) *State:* Our parameters of interest are p_1 = true proportion of young blacks who listen to rap music every day and p_2 = true proportion of young whites who listen to rap music every day. We want to estimate the difference $p_1 - p_2$ at a 95% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: $n_1 = 634$ is less than 10% of all young blacks and $n_2 = 567$ is less than 10% of all young whites. Large Counts: The number of successes and failures in both groups are at least 10 (Young blacks: 368 successes, 266 failures. Young whites: 130 successes, 437 failures). *Do:* The 95% confidence interval is

$$(0.580-0.229)\pm 1.96\sqrt{\frac{0.580(0.420)}{634}+\frac{0.229(0.771)}{567}} = 0.351 \pm 0.052 = (0.299, 0.403).$$ *Conclude:*

We are 95% confident that the interval from 0.299 to 0.403 captures the true difference in the proportions of young blacks and young whites who listen to rap music every day.

10.11 (a) *State:* Our parameters of interest are p_1 = true proportion of young men who live in their parents' home and p_2 = true proportion of young women who live in their parents' home. We want to estimate the difference $p_1 - p_2$ at a 99% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. Random: Even though the data came from a single random sample, it is reasonable to consider the two samples independent because knowing the response of a male shouldn't help us predict the response of a female. 10%: n_1 = 2253 is less than 10% of the population of young men and n_2 = 2629 is less than 10% of the population of young women. Large Counts: The number of successes and failures in both groups are at least 10 (Young men: 986 successes, 1267 failures. Young women: 923 successes, 1706 failures). *Do:* From the data we find that $n_1 = 2253$, $\hat{p}_1 = \frac{986}{2253} = 0.438$, $n_2 = 2629$, and $\hat{p}_2 = \frac{923}{2629} = 0.351$. The 99% confidence interval is

$$\left(0.438-0.351\right)\pm 2.576\sqrt{\frac{0.438\left(0.562\right)}{2253}+\frac{0.351\left(0.649\right)}{2629}} = 0.087 \pm 0.036 = \left(0.051, 0.123\right).$$

Conclude: We are 99% confident that the interval from 0.051 to 0.123 captures the true difference in the proportions of young men and young women who live in their parents' home.
(b) Because the interval does not contain 0, there is convincing evidence that the true proportion of young men who live at their parents' home is different than the true proportion of young women who live in their parents' home.

10.12 (a) *State:* Our parameters of interest are p_1 = true proportion of older black men in Atlantic City who feel vulnerable to crime and p_2 = true proportion of older black women in Atlantic City who feel vulnerable to crime. We want to estimate the difference $p_1 - p_2$ at a 90% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: $n_1 = 63$ is less than 10% of all older black men from Atlantic City and $n_2 = 56$ is less than 10% of all older black women from Atlantic City. Large Counts: The number of successes and failures in both groups are at least 10 (Older black men: 46 successes, 17 failures. Older black women: 27 successes, 29 failures). *Do:* From the data we find that $n_1 = 63,\ \hat{p}_1 = \frac{46}{63} = 0.730,\ n_2 = 56,$ and $\hat{p}_2 = \frac{27}{56} = 0.482$. So our 90% confidence interval is $(0.730 - 0.482) \pm 1.645\sqrt{\frac{0.730(0.270)}{63} + \frac{0.482(0.518)}{56}} = 0.248 \pm 0.143 = (0.105, 0.391)$. *Conclude:* We are 90% confident that the interval from 0.105 to 0.391 captures the true difference in the proportions of older black men and older black women who feel vulnerable to crime.
(b) Because the interval does not contain 0, there is convincing evidence that the true proportion of older black men who feel vulnerable to crime is different than the true proportion of older black women who feel vulnerable to crime.

10.13 $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 \neq 0$ where p_1 is the true proportion of all teens who would say that they own an iPod or MP3 player and p_2 is the true proportion of all young adults who would say that they own an iPod or MP3 player.

10.14 $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 \neq 0$ where p_1 is the true proportion of high school freshmen in Illinois who have used anabolic steroids and p_2 is the true proportion of high school seniors in Illinois who have used anabolic steroids.

10.15 *State:* We want to perform a test at the $\alpha = 0.05$ significance level of the hypotheses stated in Exercise 13. *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: $n_1 = 800$ is less than 10% of all teens and $n_2 = 400$ is less than 10% of all young adults. Large Counts: The number of successes and failures in both groups are at least 10 (Teens: 632 successes and 168 failures. Young adults: 268 successes and 132 failures.) *Do:* The proportions of those owning iPods or MP3 players in each group are $\hat{p}_1 = \frac{632}{800} = 0.79$ and $\hat{p}_2 = \frac{268}{400} = 0.67$. The pooled proportion is $\hat{p}_C = \frac{632 + 268}{800 + 400} = \frac{900}{1200} = 0.75$. The test statistic is $z = \frac{(0.79 - 0.67) - 0}{\sqrt{\frac{(0.75)(0.25)}{800} + \frac{(0.75)(0.25)}{400}}} = 4.53$ and the *P*-value is $2P(Z \geq 4.53) \approx 0$. *Conclude:* Because the *P*-value of close to 0 is less than α = 0.05, we reject H_0. There is convincing evidence that the true proportions of teens who would say that they own an iPod or MP3 player is different than the true proportions of young adults who would say that they own an iPod or MP3 player.

10.16 (a) *State:* We want to perform a test at the $\alpha = 0.05$ significance level of the hypotheses stated in Exercise 14. *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: n_1 = 1679 is less than 10% of high school freshmen in Illinois and n_2 = 1366 is less than 10% of high school seniors in Illinois. Large Counts: The number of successes and failures in both groups are at least 10 (Freshmen: 34 successes, 1645 failures. Seniors: 24 successes, 1342 failures). *Do:* The proportions of those using anabolic steroids in each group are $\hat{p}_1 = \frac{34}{1679} = 0.0203$ and $\hat{p}_2 = \frac{24}{1366} = 0.0176$. The pooled proportion is $\hat{p}_C = \frac{34+24}{1679+1366} = \frac{58}{3045} = 0.0190$. The test statistic is $z = \frac{(0.0203 - 0.0176) - 0}{\sqrt{\frac{(0.019)(0.981)}{1679} + \frac{(0.019)(0.981)}{1366}}} = 0.54$ and the *P*-value is $2P(Z \geq 0.54) = 2(0.2946) = 0.5892$. *Conclude:* Because the *P*-value of 0.5892 is greater than α = 0.05, we fail to reject H_0. We do not have convincing evidence that there is a difference in the true proportions of all Illinois freshmen and seniors who have used anabolic steroids.

10.17 *State:* Our parameters of interest are p_1 = the true proportion of all teens who would say that they own an iPod or MP3 player and p_2 = the true proportion of all young adults who would say that they own an iPod or MP3 player. We want to estimate the difference $p_1 - p_2$ at a 95% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. The conditions are met (see Exercise 10.15). *Do:* From the data we find that $n_1 = 800$, $\hat{p}_1 = 0.79$, $n_2 = 400$, and $\hat{p}_2 = 0.67$. The 95% confidence interval is $(0.79 - 0.67) \pm 1.96\sqrt{\frac{0.79(0.21)}{800} + \frac{0.67(0.33)}{400}} = 0.12 \pm 0.054 = (0.066, 0.174)$. *Conclude:* We are 95% confident that the interval from 0.066 to 0.174 captures the true difference in proportions of teens and young adults who own iPods or MP3 players. Because 0 is not included in the confidence interval and we rejected $H_0: p_1 - p_2 = 0$, the interval is consistent with the results of Exercise 15. In both cases we can rule out 0 as a plausible value for the difference in proportions.

10.18 *State:* Our parameters of interest are p_1 = the true proportion of high school freshman in Illinois who have used anabolic steroids and p_2 = the true proportion of high school seniors in Illinois who have used anabolic steroids. We want to estimate the difference $p_1 - p_2$ at a 95% confidence level. *Plan:* We should use a two-sample *z* interval for $p_1 - p_2$ if the conditions are met. The conditions are met (see Exercise 10.16). *Do:* From the data we find that $n_1 = 1679$, $\hat{p}_1 = 0.0203$, $n_2 = 1366$, and $\hat{p}_2 = 0.0176$. The 95% confidence interval is $(0.0203 - 0.0176) \pm 1.96\sqrt{\frac{0.0203(0.9797)}{1679} + \frac{0.0176(0.9824)}{1366}}$ = 0.0027 ± 0.0097 = (–0.007, 0.0124). *Conclude:* We are 95% confident that the interval from –0.007 to 0.0124 captures the true difference in the proportions of all Illinois freshmen and seniors who have used anabolic steroids. Because 0 is included in the confidence interval and we failed to reject $H_0: p_1 - p_2 = 0$, the interval is consistent with the results of Exercise 16. In both cases 0 is a plausible value for the difference in proportions.

10.19 *State:* We want to perform a test at the $\alpha = 0.05$ significance level of $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ where p_1 is the true proportion of 6- to 7-year-olds who would sort correctly and p_2 is the true proportion of 4- to 5-year-olds who would sort correctly. *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from independent random samples. 10%: $n_1 = 53$ is less than 10% of all 6- to 7-year olds and $n_2 = 50$ is less than 10% of all 4- to 5-year-olds. Large Counts: The number of successes and failures in both groups are at least 10 (6- to 7-year-olds: 28 successes, 25 failures. 4- to 5-year-olds: 10 successes, 40 failures.). *Do:* The proportions who sorted correctly in each group are $\hat{p}_1 = \frac{28}{53} = 0.528$ and $\hat{p}_2 = \frac{10}{50} = 0.20$. The pooled proportion is $\hat{p}_C = \frac{28+10}{53+50} = \frac{38}{103} = 0.369$. The test statistic is $z = \frac{(0.528 - 0.20) - 0}{\sqrt{\frac{(0.369)(0.631)}{53} + \frac{(0.369)(0.631)}{50}}} = 3.45$ and the *P*-value is $P(Z \geq 3.45) = 0.0003$. *Conclude:* Because the *P*-value of 0.0003 is less than $\alpha = 0.05$, we reject H_0. We have convincing evidence that the true proportion of 6- to 7-year-olds who would sort correctly is greater than the true proportion of 4- to 5-year-olds who would sort correctly.

10.20 *State:* We want to perform a test at the $\alpha = 0.05$ significance level of $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ where p_1 is the true proportion of male, black, never-married college students who would say "yes" and p_2 is the true proportion of female, black, never-married college students who would say "yes." *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: Even though the data came from a single random sample, it is reasonable to consider the two samples independent because knowing the response of a male shouldn't help us predict the response of a female. 10%: $n_1 = 149$ is less than 10% of all male, black, never-married college students and $n_2 = 236$ is less than 10% of all female, black, never-married college students. Large Counts: The number of successes and failures in both groups are at least 10 (Males: 91 successes, 58 failures. Females: 117 successes, 119 failures). *Do:* The proportions who said "yes" in each group are $\hat{p}_1 = \frac{91}{149} = 0.611$ and $\hat{p}_2 = \frac{117}{236} = 0.496$. The pooled proportion is $\hat{p}_C = \frac{91+117}{149+236} = \frac{208}{385} = 0.540$. The test statistic is

$$z = \frac{(0.611 - 0.496) - 0}{\sqrt{\frac{(0.540)(0.460)}{149} + \frac{(0.540)(0.460)}{236}}} = 2.21$$ and the *P*-value is $P(Z \geq 2.21) = 0.0136$.

Conclude: Because the *P*-value of 0.0136 is less than $\alpha = 0.05$, we reject H_0. We have convincing evidence to conclude that true proportion of male, black, never-married college students who would say "yes" is greater than the true proportion of female, black, never-married college students who would say "yes" to the question "Would you marry a person from a lower social class than your own?"

10.21 (a) *State*: We want to perform a test of $H_0: p_A - p_B = 0$ versus $H_a: p_A - p_B > 0$ where p_A is the true proportion of students like these who would pass the driver's license exam when taught by instructor A and p_B is the true proportion of students like these who would pass the driver's license exam when taught by instructor B. We'll use $\alpha = 0.05$. *Plan*: If conditions are met, we should do a two-sample *z* test for $p_A - p_B$. Random: The data come from two groups in a randomized experiment. Large Counts: The number of successes and failures in each group is at least 10 (A: 30 successes, 20 failures. B: 22 successes, 28 failures.) *Do:* The proportion of those who passed with instructor A is $\hat{p}_A = \frac{30}{50} = 0.60$ and with instructor B is $\hat{p}_B = \frac{22}{50} = 0.44$.

The pooled proportion is $\hat{p}_C = \frac{30+22}{50+50} = \frac{52}{100} = 0.52$. The test statistic is

$z = \frac{(0.60-0.44)-0}{\sqrt{\frac{(0.52)(0.48)}{50}+\frac{(0.52)(0.48)}{50}}} = 1.60$ and the *P*-value is $P(Z \geq 1.60) = 0.0547$. *Conclude:*

Because the *P*-value of 0.0547 is greater than $\alpha = 0.05$, we fail to reject H_0. There is not convincing evidence that the true proportion of students like these who would pass using instructor A is greater than the true proportion who would pass using instructor B.

(b) A Type I error is finding convincing evidence that instructor A is more effective than instructor B, when in reality the instructors are equally effective. A Type II error is not finding convincing evidence that instructor A is better, when in reality instructor A is more effective. Because we didn't find convincing evidence that instructor A is more effective, it is possible we made a Type II error.

10.22 (a) *State:* We want to perform a test at the $\alpha = 0.05$ significance level of $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ where p_1 is the true proportion of patients like the ones in the study who would have a stroke when taking aspirin alone and p_2 is the true proportion of patients like the ones in the study who would have had a stroke when taking both drugs. *Plan:* We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are satisfied. Random: The data come from two groups in a randomized experiment. Large Counts: The number of successes and failures in both groups are at least 10 (Aspirin alone: 206 successes, 1443 failures. Additional medicine group: 157 successes, 1493 failures). *Do:* The proportions of stroke victims in each group are $\hat{p}_1 = \frac{206}{1649} = 0.125$ and $\hat{p}_2 = \frac{157}{1650} = 0.095$. The pooled proportion is

$\hat{p}_C = \frac{206+157}{1649+1650} = \frac{363}{3299} = 0.11$. The test statistic is $z = \frac{(0.125-0.095)-0}{\sqrt{\frac{(0.11)(0.89)}{1649}+\frac{(0.11)(0.89)}{1650}}} = 2.75$ and the *P*-value is $P(Z \geq 2.75) = 0.0030$. *Using technology: z* = 2.73 and *P*-value = 0.0031.

Conclude: Because the *P*-value of 0.0031 is less than $\alpha = 0.05$, we reject H_0. We have convincing evidence that the true proportion of patients like the ones in the study who would have a stroke when taking aspirin alone is greater than the true proportion of patients like the ones in the study who would have had a stroke when taking both drugs. In other words, we have convincing evidence that adding dipyridamole helps reduce the risk of stroke.

(b) A Type I error is finding convincing evidence that adding dipyridamole helps prevent strokes, when in reality adding dipyridamole doesn't help. A Type II error is not finding convincing evidence that adding dipyridamole helps prevent strokes, when in reality adding dipyridamole does help. A Type II error would be more serious in this case because we would be missing out on a drug that could be preventing people from having a stroke. The consequence of making a Type I error is the extra cost of a drug that isn't any more effective.

10.23 (a) We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from two groups in a randomized experiment. Large Counts: The number of successes and failures in both groups are at least 10 (Prayer group: 44 successes, 44 failures. Control group: 21 successes, 60 failures).
(b) If there is no difference in the true pregnancy rates of women who are being prayed for and those who are not, there is a 0.0007 probability of getting a difference in pregnancy rates as large or larger than the one observed in the experiment (0.500 – 0.259 = 0.241) by chance alone.
(c) Because the *P*-value of 0.0007 is less than α = 0.05, we reject H_0. There is convincing evidence that the pregnancy rates among women like these who are prayed for is higher than the pregnancy rates for those who are not prayed for.
(d) It is important that the women who were trying to get pregnant didn't know whether they were being prayed for or not. This knowledge might have affected their behavior in some way (even unconsciously) that would have affected whether they became pregnant or not. Then, we wouldn't know if it was the prayer or the other behaviors that caused the higher pregnancy rate.

10.24 (a) We should use a two-sample *z* test for $p_1 - p_2$ if the conditions are met. Random: The data come from two groups in a randomized experiment. Large Counts: The number of successes and failures in both groups are at least 10 (Acupuncture group: 34 successes, 46 failures. Control group: 21 successes, 59 failures).
(b) If there is no difference in pregnancy rates of women who receive acupuncture and those who don't, there is a 0.0152 probability of getting a difference in pregnancy rates as large or larger than the one observed in the experiment (0.425 – 0.2625 = 0.1625) by chance alone.
(c) Because the *P*-value of 0.0152 is less than α = 0.05, we reject H_0. We have convincing evidence that the pregnancy rate among women like these who receive acupuncture is higher than the pregnancy rate for those who do not receive acupuncture.
(d) Knowing whether or not they received acupuncture might have affected the women's behavior in some way (even unconsciously) that would have affected whether they became pregnant or not. Then, we wouldn't know if it was the acupuncture or the other behaviors that caused the higher pregnancy rate.

10.25 a

10.26 e

10.27 c

10.28 e

10.29 (a) $\hat{y} = -13{,}832 + 14{,}954x$ where $\hat{y}$ = the predicted mileage and x = the age in years of the cars.
(b) For each year older the car is, the predicted mileage will increase by 14,954 miles.

(c) For a 10-year-old car, the predicted mileage is $\hat{y} = -13{,}832 + 14{,}954(10) = 135{,}708$. The residual is actual mileage – predicted mileage = $110{,}000 - 135{,}708 = -25{,}708$ miles. The student's car had 25,708 fewer miles than expected, based on its age.

10.30 (a) 77% of the variation in mileage is accounted for by the linear model relating mileage to age.
(b) The least-squares regression line always goes through the point $(\bar{x}, \bar{y})$, so $\bar{y} = -13{,}832 + 14{,}954\bar{x} = -13{,}832 + 14{,}954(8) = 105{,}800$ miles.
(c) When using the least-squares regression line with x = age to predict y = mileage, we will typically be off by about 22,723 miles.
(d) No, it would not be reasonable to use the least-squares line to predict a car's mileage from its age for a teacher. The least-squares line is based on a sample of cars owned and driven by students, not teachers.